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Pressure dependence of acoustic velocities and the Lamé-coefficients

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Introduction

Unconventional reservoir

↓

Hydraulic fracturing

↓

Stress *(Anisotropy)*

→ Acoustic velocity

→ Elastic moduli

Porosity

Permeability

etc.

?

Pressure

Literature review

- Birch (1960)
- Brace & Walsh (1964)
- Wepfer & Christensen (1991)
- Wang et al. (2007)

- Qualitative models
- No physical explanation
- Function parameters using mathematical regression methods

- Somogyi
  - New r… …ure dependence of longitudinal acoustic wave velocity

$$V(P) = AP^{a} + B(1 - \exp(-bP))$$

$$V(P) = a(\ln P)^{2} + b \ln P + c$$

V – velocity
P – pressure
a,b,c – constants

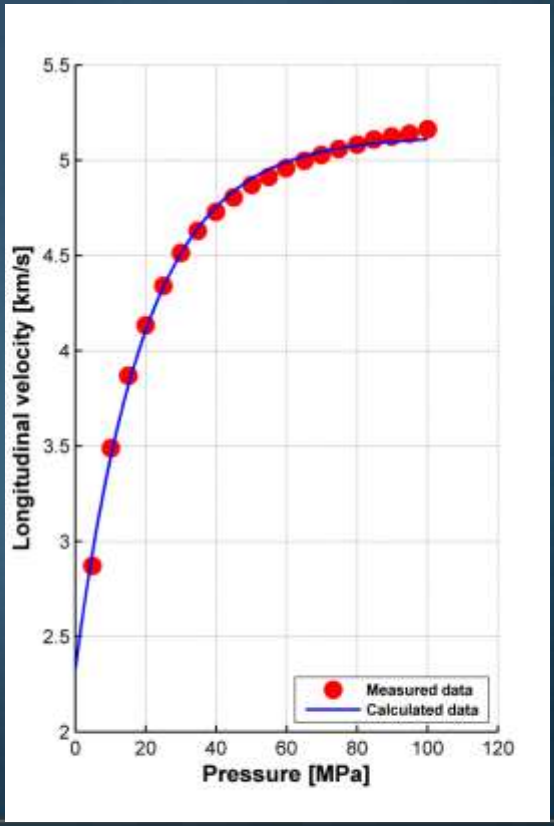
Rock physical model for σ-v_p

(Somogyi Molnár, 2013)

- Basic consideration: Birch (1960)

  The main factor in pressure dependence of propagation velocity is the closure of pores.

- Limits:
  - Only for uniaxial stress state
  - Valid in the reversible domain of the medium



The model (Somogyi Molnár, 2013)

$$\boxed{dV = -\lambda_V V d\sigma} \quad V = V_0 \exp(-\lambda_V \sigma)$$

$$\boxed{d\alpha = -\kappa_p dV}$$

$$d\alpha = -\kappa_p \lambda_V V_0 \exp(-\lambda_V \sigma) d\sigma$$

$$\alpha = K - \kappa_p V_0 \exp(-\lambda_V \sigma)$$

$$\alpha_0 = K - \kappa_p V_0 \qquad \Delta\alpha_0 = \kappa_p V_0$$

- V – pore volume
- σ – stress
- dV – change in V_{pore}
- $d\sigma$ – stress increase
- V_0 – V_{pore} at σ_0
- α – P-wave velocity
- κ_p – material characteristic
- K – integration constant
- α_0 – α at σ_0
- $\Delta\alpha_0$ – velocity-drop ($\alpha_{max} - \alpha_0$)
- λ_v – petrophysical constant

$$\boxed{\alpha = \alpha_0 + \Delta\alpha_0(1 - \exp(-\lambda_V \sigma))}$$

Valid also for shear waves (β)?

- Different properties – different behaviour
- Tested on literature data sets
- Determination of the pressure dependence of P- and S-wave velocities in two independent inversion procedure

$$\alpha = \alpha_0 + \Delta\alpha_0 (1 - \exp(-\lambda_V \sigma))$$

$$\beta = \beta_0 + \Delta\beta_0 (1 - \exp(-\lambda_V \sigma))$$

- β_0 – transversal wave velocity at stress-free state (σ_0)
- $\Delta\beta_0$ – velocity-drop ($\beta_{max} - \beta_0$)
- λ_v – petrophysical constant

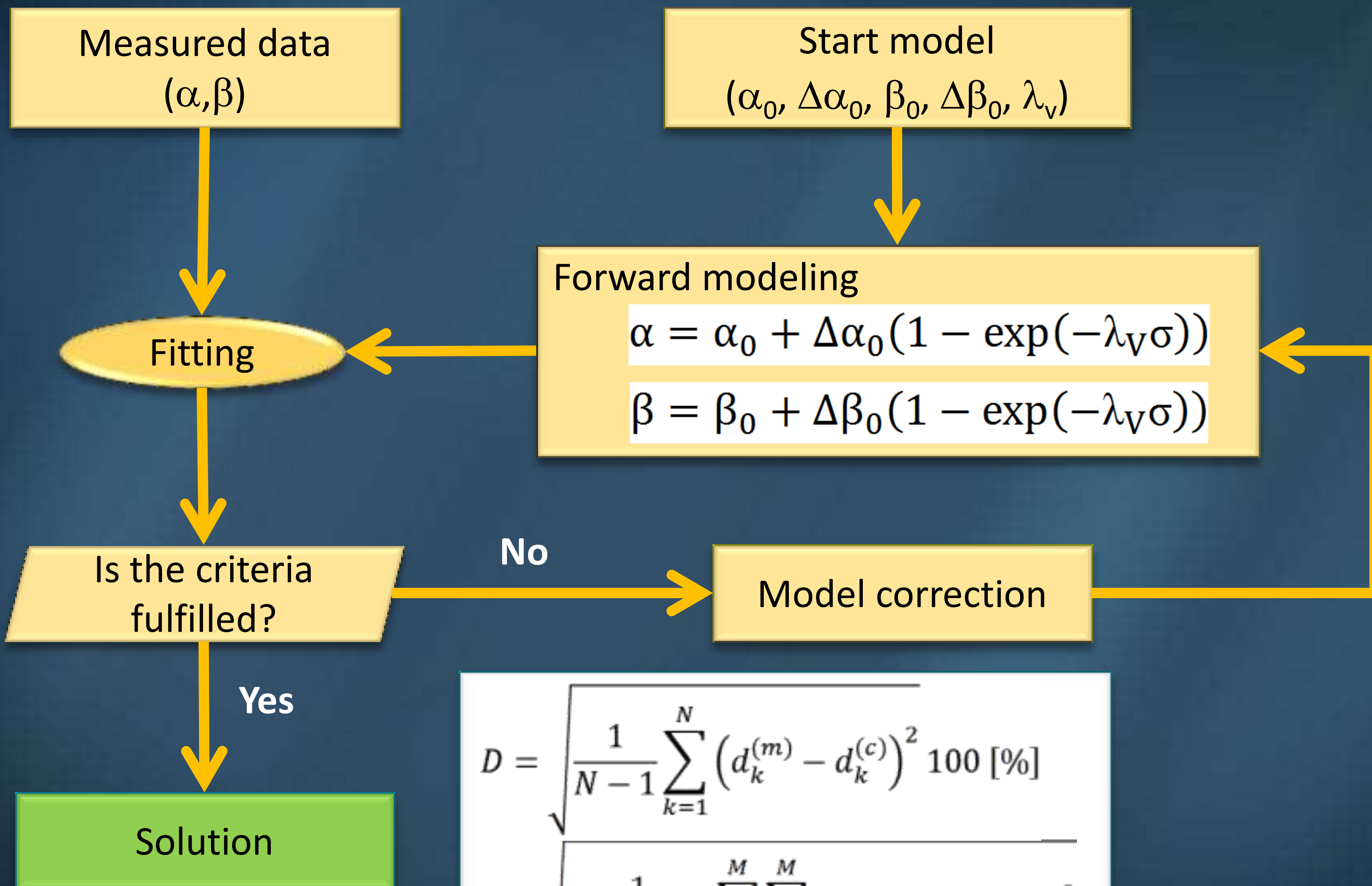
Examination of λ_v

- Determination of λ_v in distinct inversion procedures

	λ_v (P-wave) [1/MPa]	λ_v (S-wave) [1/MPa]
Conglomerate	0.0504	0.0515
Berea sandstone	0.1470	0.1293

- Same λ_v → common parameter
- Joint inversion
  - 5 model parameters
  - α_0, $\Delta\alpha_0$, β_0, $\Delta\beta_0$, λ_v

Joint inversion procedure



$$D = \sqrt{\frac{1}{N-1}\sum_{k=1}^{N}\left(d_k^{(m)} - d_k^{(c)}\right)^2} \; 100\ [\%]$$

$$S = \sqrt{\frac{1}{M(M-1)}\sum_{i=1}^{M}\sum_{j=1}^{M}\left(corr(m)_{ij} - \delta_{ij}\right)^2}$$

Lamé-coefficients (μ, λ)

- Perfectly elastic medium
- Locally interpretable quantities in the given deformation or rather stress state
  - μ - 1. Lamé-parameter (=G=shear modulus)
  - λ - 2. Lamé-parameter
- Relationship to the acoustic velocities

  ρ – density of the medium

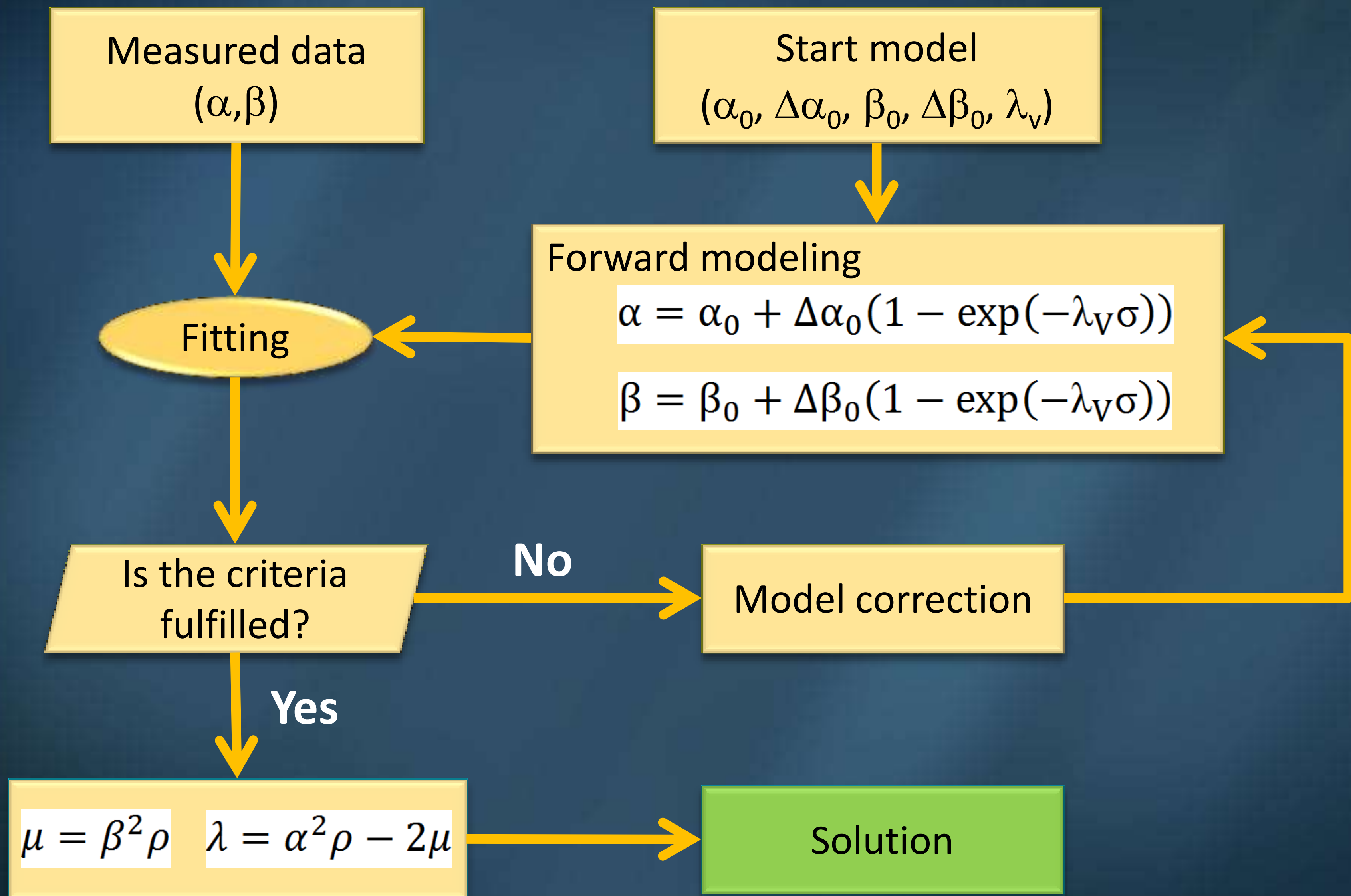
$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} \qquad \beta = \sqrt{\frac{\mu}{\rho}}$$

$$\boxed{\mu = \beta^2 \rho} \qquad \boxed{\lambda = \alpha^2 \rho - 2\mu}$$

$$K = \lambda + \frac{2\mu}{3} \qquad G = \mu$$

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu} \qquad m = \frac{1}{\nu} = \frac{2(\lambda + \mu)}{\lambda}$$

Joint inversion for Lamé-coefficients



Case study – Samples

He & Schmitt (2003)

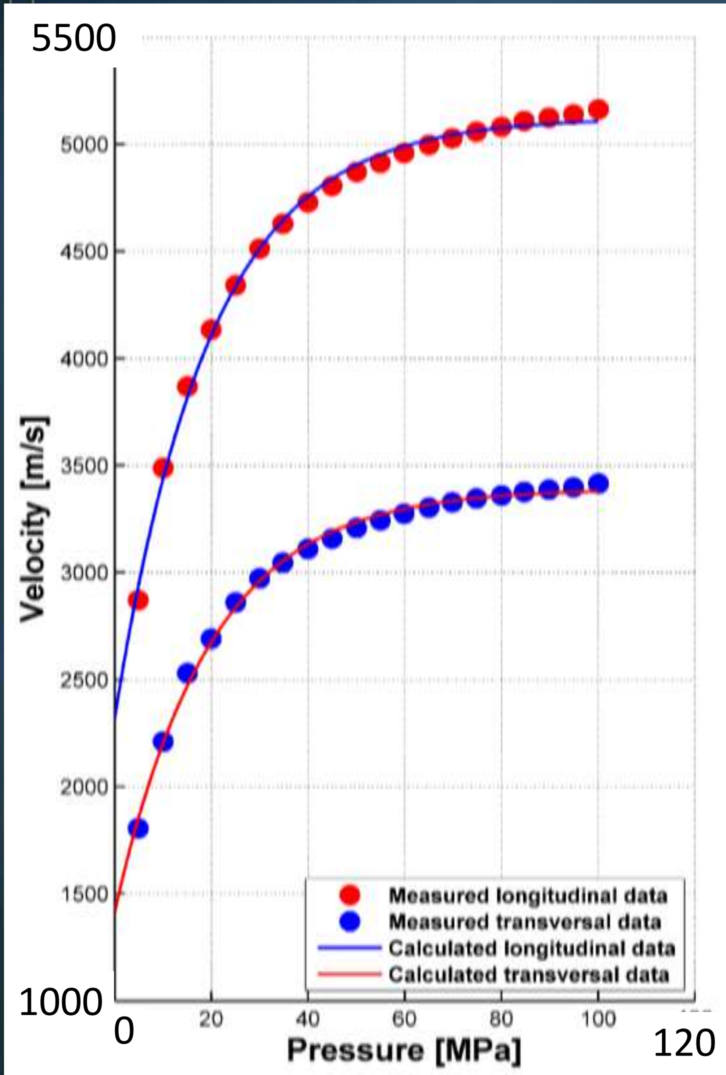
- Conglomerate
- Dry
- Bulk density = 2300 kg/m^3
- Low porosity

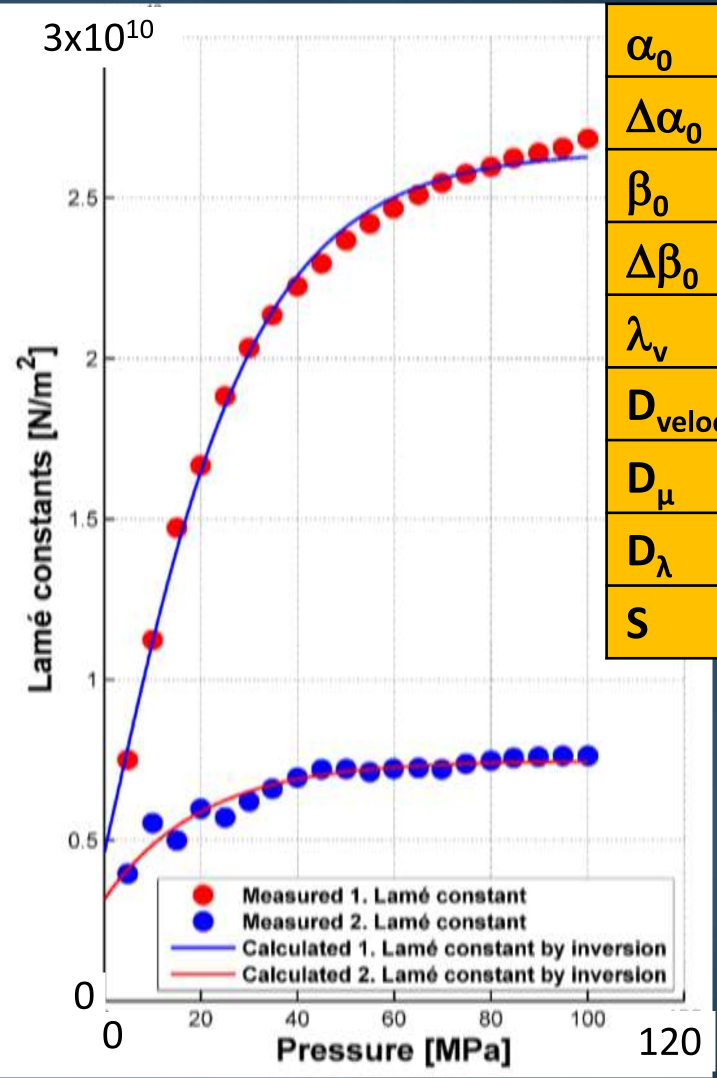
Winkler & Murphy (1995)

- Berea Sandstone
- Dry
- Bulk density ≈ 2600 kg/m^3
- Porosity ≈ 16 %

- Measurement of acoustic velocities
  - Uniaxial load
  - Transducer & receiver on end caps
  - velocity = sample length / travel time

Results - Conglomerate

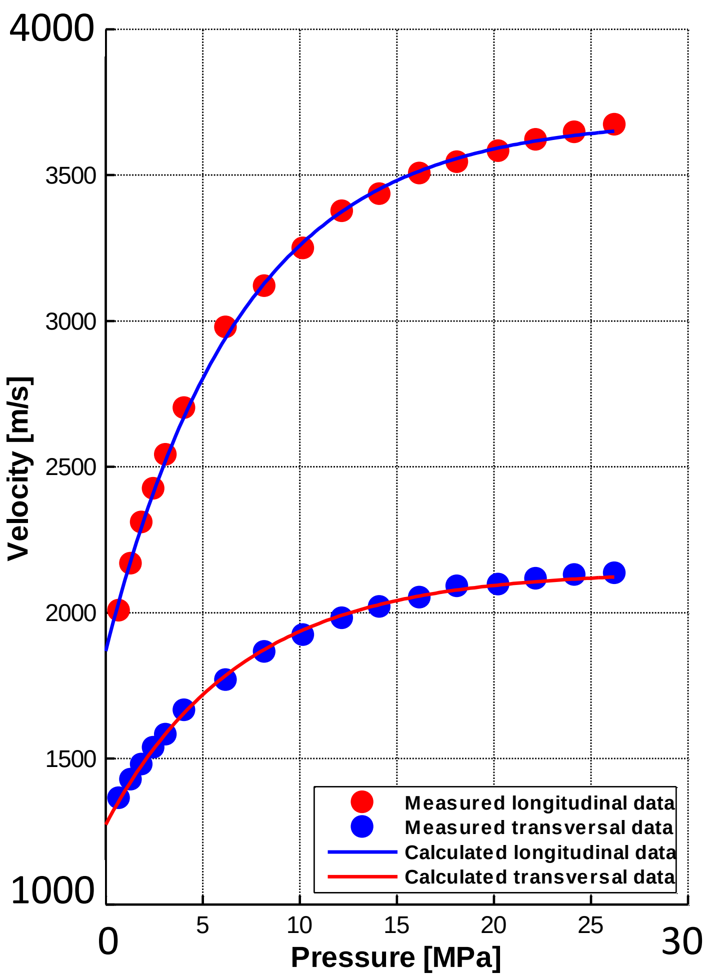


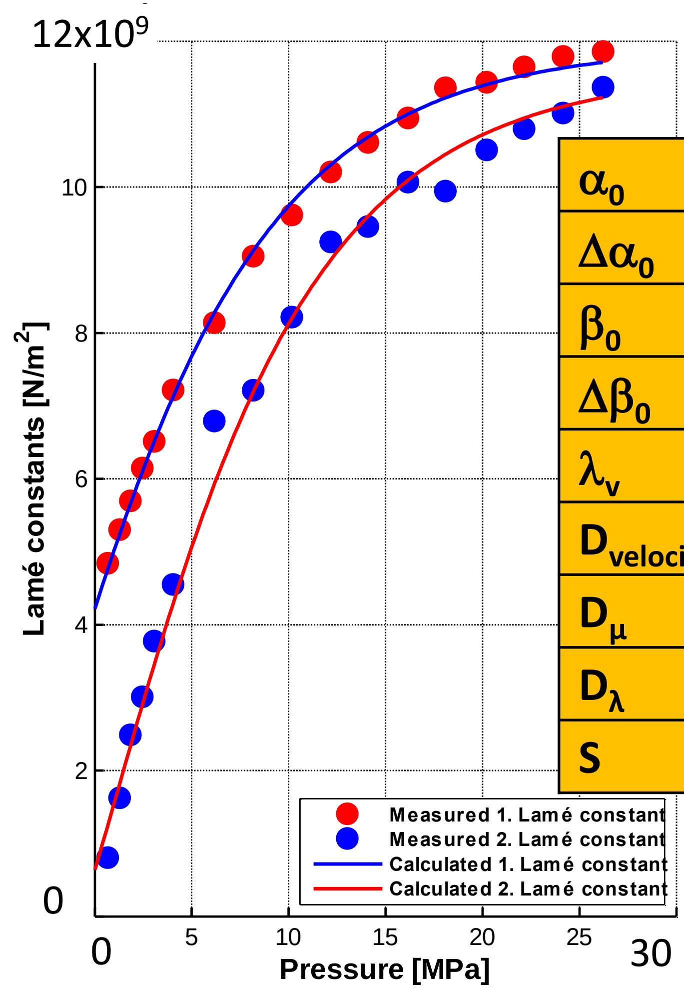


α_0	2323.2 [m/s]
$\Delta\alpha_0$	2801.0 [m/s]
β_0	1418.2 [m/s]
$\Delta\beta_0$	1973.4 [m/s]
λ_v	0.0508 [1/MPa]
$D_{velocity}$	2.90 %
D_{μ}	3.25 %
D_{λ}	2.52 %
S	0.60

λ_v (P-wave)
0.0504 [1/MPa]
λ_v (S-wave)
0.0515 [1/MPa]

Results – Berea sandstone





α_0	1867.8 [m/s]
$\Delta\alpha_0$	1825.0 [m/s]
β_0	1273.5 [m/s]
$\Delta\beta_0$	868.8 [m/s]
λ_v	0.1436 [1/MPa]
$D_{velocity}$	1.48%
D_{μ}	1.01 %
D_{λ}	3.06 %
S	0.44

λ_v (P-wave)
0.1470 [1/MPa]
λ_v (S-wave)
0.1293 [1/MPa]

Summary

- New model for describing the pressure dependence of shear wave velocity based on the rock physical model developed for the longitudinal wave velocity

$$\beta = \beta_0 + \Delta\beta_0(1 - \exp(-\lambda_V \sigma))$$

- Determination of stress-dependent Lamé-coefficients

- Joint inversion
  - 5 model parameters
  - α_0 – longitudinal wave velocity at stress-free state (σ_0)
  - $\Delta\alpha_0$ – velocity-drops ($\alpha_{max} - \alpha_0$)
  - β_0 – transversal wave velocity at stress-free state (σ_0)
  - $\Delta\beta_0$ – velocity-drops ($\beta_{max} - \beta_0$)
  - λ_v – common petrophysical constant

- Good correlation between laboratory measured and calculated data

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